PHYS 242 BLOCK 3 NOTES

Sections 23.1 to 23.5

Rewriting General Physics I's Eq. (6.14) as Eq. (23.1), the work $W_{a\to b}$ done by a force $\vec{F}$ in moving a particle from point a to point b is defined to be $W_{a\to b} \equiv \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F \cos\phi \, dl$.

If the force is conservative, $W_{a\to b} = U_a - U_b$, where U is the potential energy associated with that force.

If we have a test charge q_0 outside a spherically-symmetric, same-sign charge q, $W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b F\cos\phi\, dl$, where $F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$ and $dl\cos\phi = dr$ (see Fig. 23.6).Then integrating from r_a to r_b gives us $\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r_a} - \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r_b} = U_a - U_b$. Thus the simplest choice for the potential energy expression is $\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}}$.

This equation also holds for opposite-sign charges. Any equation in this block containing ε_0 is for vacuum ≈ air.

U is the **electric potential energy** (in J) (TERM 1).

Recall that $\frac{1}{4\pi\varepsilon_0} = 8.988 \times 10^9 \frac{\text{N·m}^2}{\text{C}^2} \approx 9.0 \times 10^9 \frac{\text{N·m}^2}{\text{C}^2}$.

q and q_0 are the charges (in C)—(point or spherically symmetric) (recall that charges can be +, 0, or –).

r is the center-to-center distance (in m) between the two charges (recall that r is *always* +).

Thus we see that U is positive for same-sign charges and negative for opposite-sign charges.

We also see that U approaches zero as the distance r approaches infinity.

This U can be used in $K_a + U_a = K_b + U_b$ ($W_{\text{other}} = 0$) (SKILL 1), as in Example 23.1.

The **(electric) potential** V at a point is the electric potential energy per charge at that point, $V \equiv \frac{U}{q_0}$ (TERM 2) . Therefore, dividing both sides of $W_{a\to b} = U_a - U_b$ by q_0 tells us that the work per charge $\frac{W_{a\to b}}{q_0}$ is the **(electric) potential difference** $V_a - V_b$ between points a and b (TERM 3): $\boxed{\frac{W_{a\to b}}{q_0} = V_a - V_b}$. We often shorten $V_a - V_b$ to V_{ab} (and for electric circuits, often even further–and confusingly–to just V).

V is the (electric) potential (in V = volt = $\frac{\text{joule}}{\text{coulomb}} = \frac{\text{J}}{\text{C}}$) ($V$ can be +, 0, or –). Please don't mix up U, V, and V.

$W_{a\to b}$ is the work done (in J) by the external electric field in moving q_0 from a to b ($W_{a\to b}$ can be +, 0, or –).

Note that the two bold-face statements on page 759 (next to Fig. 23.11) are wrong because V ≠ J (units).

For a single point charge q (or outside a spherically symmetric charge q) we can substitute $U = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$ to find $\boxed{V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$. For a collection of these charges, $\boxed{V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots\right)$.

q_i is the charge (in C) of object i (q_i is +, 0, or –).

r_i is the distance (in m) from the center of object i (r_i is always +).

For a continuous charge distribution, $\boxed{V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}}$. That is, first find an expression for the dV from an arbitrary dq in the distribution, then integrate to find V (SKILL 2).

Finding $V_a - V_b$ *from* $\vec{E}$: Recall that $V_a - V_b = \frac{W_{a \to b}}{q_0} = \frac{\int_a^b \vec{F} \cdot d\vec{l}}{q_0} = \frac{\int_a^b q_0 \vec{E} \cdot d\vec{l}}{q_0}$. Canceling out q_0, we have an equation we can use to find the potential difference *if* we know $\vec{E}$ at all points along *any* path from a to b.

$$\boxed{V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E \cos \phi \, dl}.$$

In this introductory course, we'll replace dl with dx or dy or dz or dr in the integral.

Note that $\vec{E}$ can have units of either $\frac{\text{V}}{\text{m}}$ or $\frac{\text{N}}{\text{C}}$ because $1\,\frac{\text{V}}{\text{m}} = 1\,\frac{\text{J/C}}{\text{m}} = 1\,\frac{\text{N}\cdot\cancel{\text{m}}/\text{C}}{\cancel{\text{m}}} = 1\,\frac{\text{N}}{\text{C}}$.

Now suppose that an electric field moves an electron (charge = $-e = -1.602 \times 10^{-19}$ C) through a potential *rise* of one volt (so V_b is *larger* than V_a by 1 V). Then

$$W_{a \to b} = q_0(V_a - V_b) = -e(-1\text{ V}) = 1\text{ eV} = (1.602 \times 10^{-19}\,\cancel{\text{C}})(1\text{ J}/\cancel{\text{C}}) = 1.602 \times 10^{-19}\text{ J}.$$

That is, one **electron volt** (1 eV) is defined to equal 1.602×10^{-19} J of work or energy (TERM 4).

Cover up the solutions and carefully work Examples 23.3 to 23.11.

The term *equipotential* means constant potential: V is constant at all points along an **equipotential line**, over an **equipotential surface**, or throughout an **equipotential volume** (TERMS 5, 6, and 7). For equipotential lines or surfaces, $V_a - V_b = V - V = 0 = \int_a^b E \cos \phi \, dl$ tells us that ϕ must be 90° (if $E \neq 0$). Thus, **electric field lines are always perpendicular to equipotential lines or equipotential surfaces.** If its free charges are at rest overall, a conducting surface is always an equipotential surface. Therefore, any electric field at its surface will be normal to that surface. Since the derivative of a constant is zero, $\vec{E} = -\vec{\nabla} V$ (see below) tells us **throughout an equipotential volume, $E = 0$.**

The electric field from a charged conducting surface tends to be greatest where the radius of curvature is smallest (for sharp points and thin wires). This large electric field can break down the air, giving an electrical discharge. In contrast, flatter areas tend to have smaller surface electric fields (SKILL 3).

Finding $\vec{E}$ *from* V: $V_a - V_b = \int_b^a dV = \int_a^b (-dV)$ and $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$ tells us that $-dV = \vec{E} \cdot d\vec{l}$.

Evaluating the scalar (dot) product in terms of components (see Eq. (1.19)) gives $-dV = E_x dx + E_y dy + E_z dz$. Keeping both y and z constant gives $dy = 0$ and $dz = 0$, so $-(\frac{dV}{dx})_{y,z\ constant} = E_x$. Such a derivative is called a *partial derivative*. Thus we have: $\boxed{E_x = -\frac{\partial V}{\partial x}}$, $\boxed{E_y = -\frac{\partial V}{\partial y}}$, $\boxed{E_z = -\frac{\partial V}{\partial z}}$, and $\boxed{E_r = -\frac{\partial V}{\partial r}}$.

E_x is the x-component of the electric field $\vec{E}$ (in $\frac{\text{V}}{\text{m}}$ or $\frac{\text{N}}{\text{C}}$) (with the same idea for y, z, or r).

Then $\vec{E} = E_x \hat{\imath} + E_y \hat{\jmath} + E_z \hat{k} = -(\hat{\imath} \frac{\partial}{\partial x} + \hat{\jmath} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z})V$. The quantity in the parentheses is the *gradient operator* $\vec{\nabla}$(in Cartesian coordinates), so $\vec{E} = -\vec{\nabla} V$. In words, the electric field equals the negative of the **potential gradient** (TERM 8).

Cover up the solutions and carefully work Examples 23.13 and 23.14. Correct the answer for 23.2 on page 784 by deleting "unit" twice and adding "per charge" after both "work" and "force".