PHYS 242 BLOCK 4 NOTES

Sections 24.1, 24.3 to 24.6

Suppose we have two conductors of any shape that are insulated from one another. Conductor a has a positive excess charge Q (in C) on it. Conductor b has a negative excess charge $-Q$ on it. This Q is called the **charge stored**. We consider only cases in which the potential difference $V_a - V_b = V_{ab}$ (in V) is directly proportional to Q. Then the ratio of Q to V_{ab} is a constant and the equation $\boxed{C \equiv \frac{Q}{V_{ab}}}$ *defines* the **capacitance** C (in F = farad = $\frac{\text{coulomb}}{\text{volt}} = \frac{\text{C}}{\text{V}}$). (Don't confuse the capacitance C with the charge unit C = coulomb.) The capacitance C is never negative. Also, the quantities Q and V_{ab} are never negative in the above defining equation.

A **capacitor** is a circuit element that mainly provides capacitance. Example 24.3 could be improved by showing $V_{ab} = V_a - V_b = \int_a^b E \cos\phi \, dl$ with $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ and $\phi = 0$ gives the V_{ab} used. Example 24.4 could be similarly improved by showing $E = \frac{\lambda}{2\pi\varepsilon_0 r}$ and $\phi = 0$ gives the V_{ab} used (as we did for b and a in class in Block 3).

For varying values, let's use υ for V_{ab} and q for Q. Then $C = \frac{Q}{V_{ab}}$ solves to $\upsilon = \frac{q}{C}$. In moving an infinitesimal charge dq from plate to plate to charge a capacitor, we do an amount of work $dW = \upsilon\, dq = \frac{q}{C}\, dq$. This work is stored as electric potential energy U. Integrating, $W = \int_0^U dW = U - 0 = \frac{1}{C}\int_0^Q q\, dq = \frac{1}{C}(\frac{Q^2}{2} - 0)$. That is, $\boxed{U = \frac{Q^2}{2C}}$. Substituting $Q = CV$ gives $\boxed{U = \frac{1}{2}CV^2}$. Finally, substituting $C = \frac{Q}{V}$ gives $\boxed{U = \frac{1}{2}QV}$, where V is short for the potential difference $V_a - V_b = V_{ab}$ (in V). This U is the electric potential energy (in J), *which is stored in the capacitor's electric field*. See if you can work Example 24.7.

The word **dielectric** is a synonym for insulator. The **dielectric strength** E_m is the maximum electric field magnitude the dielectric can withstand without breaking down and conducting.

Suppose we charge a capacitor (with vacuum between its plates and capacitance C_0) so that it has a charge stored Q, a potential difference V_0, and—at some point between the plates—an electric field magnitude E_0. If we then *completely* fill the volume between the plates of the capacitor with a dielectric and Q stays constant, we would find that the potential difference and electric field magnitude *decrease* to $V = \frac{V_0}{K}$ and $E = \frac{E_0}{K}$. Since $C = \frac{Q}{V}$, the capacitance *increases* to KC_0. The **dielectric constant** K has no unit and is greater than or equal to one ($K \geq 1$).

Why does $E = \frac{E_0}{K}$?

Step 1. The **free charges** on the *conducting* capacitor plates give an applied electric field. This field exerts a torque on the electric dipoles in the dielectric, tending to line them up, and giving induced **bound charges** on the *insulating* dielectric surfaces. (See Fig. 24.19.) (The dielectric's resulting net electric dipole moment per volume is called its **polarization**.)

Step 2. The induced bound charges give an *opposing* electric field, thus decreasing the resultant electric field magnitude from E_0 to E_0/K. (See Fig. 24.20.)

We've been using $\varepsilon_0 = 8.854 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$ or $\frac{\text{F}}{\text{m}}$, calling it the *electric constant*., but it has another name:

ε is the **permittivity** of a dielectric and then ε_0 is the *permittivity of vacuum*. The relation between ε and ε_0 is $\boxed{\varepsilon = K\varepsilon_0}$. Therefore, *in vacuum*, $\varepsilon \equiv \varepsilon_0$ by definition, so $K = 1$ there. In air, ε is slightly greater than ε_0 and K is slightly greater than 1 (for dry air at 1 atm pressure and 20˚C, $\varepsilon = 8.859 \times 10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}$ or $\frac{\mathrm{F}}{\mathrm{m}}$ and $K = 1.00059$).

An integral form of Gauss's law *for vacuum* is $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$, which gave us electric field relations for high symmetry cases. The comparable integral form of Gauss's law when we have one or more dielectrics present is $\boxed{\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\varepsilon_0}}$ or $\oint \varepsilon\vec{E} \cdot d\vec{A} = Q_{\text{encl-free}}$, where $Q_{\text{encl-free}}$ is the total free (not bound) charge enclosed by the Gaussian surface. **Thus, if the dielectric(s) keep sufficient symmetry, we can take our previous results for E and V_{ab} in vacuum and simply replace ε_0 with either ε or $K\varepsilon_0$.** The table below gives some examples for the three symmetries:

Symmetry	Spherical	Cylindrical	Flat
Vacuum	$E = \frac{1}{4\pi\varepsilon_0}\frac{\lvert q\rvert}{r^2}$	$V_{ab} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$	$E = \frac{\sigma}{\varepsilon_0}$
Dielectric	$E = \frac{1}{4\pi\varepsilon}\frac{\lvert q\rvert}{r^2}$ or $E = \frac{1}{4\pi K\varepsilon_0}\frac{\lvert q\rvert}{r^2}$	$V_{ab} = \frac{\lambda}{2\pi\varepsilon}\ln\frac{b}{a}$ or $V_{ab} = \frac{\lambda}{2\pi K\varepsilon_0}\ln\frac{b}{a}$	$E = \frac{\sigma}{\varepsilon}$ or $E = \frac{\sigma}{K\varepsilon_0}$

A **parallel-plate capacitor** has two parallel conducting plates. Each plate has an area A. The plates are separated by a distance d. We assume the dimension(s) of the plates that give us the area are much larger than d. Then E is constant between the plates (except for the small "fringing" region near the edges, which we can accurately ignore). (See Fig. 24.2). Integrating along an electric field line from positive plate a to negative plate b, we find $V_{ab} = V_a - V_b = \int_a^b E\cos\phi\, dl = E\cos 0 \int_0^d dl = E(1)d$. That is, $\boxed{V_{ab} = Ed}$. The quantities V_{ab}, E, and d are never negative in this equation. Between the parallel conducting plates: V_{ab} is the potential difference (in V), E is the magnitude of the *constant* electric field (in $\frac{\mathrm{V}}{\mathrm{m}}$ or $\frac{\mathrm{N}}{\mathrm{C}}$), and d is the distance (in m).

From the table above, $E = \frac{\sigma}{\varepsilon}$ (where $\sigma = \frac{Q}{A}$). We substitute $E = \frac{Q/A}{\varepsilon}$ into $V_{ab} = Ed$ and that result for V_{ab} into $C = \frac{Q}{V_{ab}}$ to obtain $C = \varepsilon\frac{A}{d}$. Remembering that $\varepsilon = K\varepsilon_0$, we find $\boxed{C = \varepsilon\frac{A}{d} = K\varepsilon_0\frac{A}{d}}$ *for parallel-plate capacitors only*. Cover up the solutions and carefully work Examples 24.1 (in vacuum) and 24.2, as well as most of Example 24.10 (don't worry about Q_i, σ, and σ_i).

Now we find an expression for the electric field's **energy density** u, which is its electric potential energy per volume. For an idealized parallel-plate capacitor (no electric-field fringing at its edges), $u = \frac{U}{\text{volume}}$, where $U = \frac{1}{2}CV^2$, $C = K\varepsilon_0\frac{A}{d} = \varepsilon\frac{A}{d}$, and $V = Ed$. Also the volume between the plates (the location of the electric field) equals Ad. When we make all these substitutions, we find that the parallel-plate capacitor's A and d cancel out, and we have a more general expression for the energy density u (in $\frac{\mathrm{J}}{\mathrm{m}^3}$): $\boxed{u = \frac{1}{2}K\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2}$.

Work vacuum Examples 24.8 and 24.9, as well as Example 24.11 (replacing V with *vol* in its EVALUATE part).