

Experimentally, we find $\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2}$, which has magnitude $B = \frac{\mu_0 |q| v \sin \phi}{4\pi r^2}$.

\vec{B} is the magnetic field (in T) *caused* by a moving point charge.

The **magnetic constant** μ_0 is defined to equal exactly $4\pi \times 10^{-7} \frac{T \cdot m}{A}$.

q is its charge (in C, where $1 \text{ C} \equiv 1 \text{ A} \cdot \text{s}$) and \vec{v} is its constant velocity (in $\frac{m}{s}$). Note that a nonzero \vec{B} is in the same direction as $\vec{v} \times \hat{r}$ if the charge q is positive, but opposite $\vec{v} \times \hat{r}$ if the charge q is negative.

r is the distance (in m) from the source point (the moving point charge) to the field point.

\hat{r} is a unit vector directed *from* the source point *to* the field point. Unit vectors have no SI units but have magnitude one (unity).

ϕ is the angle between the directions of \vec{v} and \hat{r} (as in Fig. 28.1a).

Cover up the solution and carefully work Example 28.1.

The **law of Biot and Savart**, $d\vec{B} = \frac{\mu_0 I d\vec{l} \times \hat{r}}{4\pi r^2}$, has magnitude $dB = \frac{\mu_0 I dl \sin \phi}{4\pi r^2}$.

To find \vec{B} , we perform the vector integral of $d\vec{B}$: $\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l} \times \hat{r}}{r^2}$.

$d\vec{B}$ is the infinitesimal magnetic field (in T) *caused* by a current I (in A) flowing in an infinitesimal length $d\vec{l}$ (in m) (as illustrated with a greatly exaggerated length labeled $d\vec{l}$ in Fig. 28.3a).

r is the distance (in m) from the source point (the infinitesimal length $d\vec{l}$) to the field point and \hat{r} is the corresponding unit vector.

ϕ is the angle between the directions of $d\vec{l}$ and \hat{r} .

All five equations above tell us that their magnetic fields are zero both directly ahead of ($\phi = 0$) and directly behind ($\phi = 180^\circ$) a moving point charge and a bit of current.

Cover up the solution and carefully work Example 28.2.

Outside a long straight wire, the law of Biot and Savart gives $B = \frac{\mu_0 I}{2\pi r}$, where r is the distance from the *center* of the wire to the field point (Roger Freedman leaves out “the center of” several times in the textbook). A long straight wire’s magnetic field lines are circles centered on the wire. **Mentally grasp the wire with your right hand, with your extended thumb in the direction the current flows. Your fingers then wrap around in the directions of \vec{B} .** (See Fig. 28.6.)

Cover up the solutions and carefully work Examples 28.3 and 28.4

From Fig. 28.9, **parallel currents attract, but antiparallel (opposite) currents repel.**

On the axis of a flat circular coil of N turns, the law of Biot and Savart gives $B = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$, where x is the distance (in m) along the axis from the center of the coil to the field point and a is the coil’s radius (in m). At

the coil's center (that is, at $x = 0$), this equation reduces to $B = \frac{\mu_0 NI}{2a}$.

Cover up the solution and carefully work Example 28.6.

On the axis of a circular coil, the direction of \vec{B} is the same direction as the coil's magnetic dipole moment and area vectors: **Wrap the fingers of your right hand around the coil the way the current flows. Then your extended right thumb points along the coil's axis in the direction of \vec{B} .**

If no magnetic materials are present, **Ampere's law** is $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$, where I_{encl} is the net *constant* current (in A) enclosed by the path of the integral. See how we use Ampere's law in the high symmetry Examples 28.7, 28.8, 28.9, and 28.10. Delete the TEST YOUR UNDERSTANDING example on page 936 and its answer on page 954 because the permanent magnet is, of course, made of a magnetic material.

A **solenoid** is a helical coil wrapped on a cylinder. When its length is much greater than its diameter, Example 28.9 shows, *near its center*, $B = \mu_0 nI$, where n is the number of turns per length (in m^{-1}). (The chapter-opening question on page 921 and its answer on page 954 are correct only near the center of solenoids whose lengths are much greater than their diameters.)

A **toroidal solenoid** (more commonly called a **toroid**) is a coil wrapped on a doughnut-shaped core. Example 28.10 shows $B = \frac{\mu_0 NI}{2\pi r}$, where N is the number of turns (no unit) and r is the distance (in m) from the center to the field point (see Fig. 28.25). This B is the magnitude of the tangential magnetic field (in T) in the "dough" of the doughnut-shaped core (that is, within the turns).

All our previous equations containing μ_0 assume any materials present to be essentially nonmagnetic.

The magnetic dipole moments of atoms are caused *mainly* by the orbital and spin motions of their electrons (nuclear magnetism is about 10^3 times smaller).

The **magnetization** \vec{M} of a material is its net magnetic dipole moment per volume.

Outside of a magnet, its own magnetic field is away from its N-pole and toward its S-pole. In general, this magnetic field decreases with distance from the magnet.

Paramagnetism is the temperature-dependent lining up of the atomic magnetic dipoles when placed in an external magnetic field. In the material (except at very low temperatures) paramagnetism gives only a slight increase over the external magnetic field value.

Diamagnetism is an induced effect that ordinarily gives a weak magnetization that opposes and slightly decreases the value of the external magnetic field in the material. It can be a strong effect in superconductors.

In **ferromagnetism**, adjacent atomic magnetic dipoles line up in strong parallelism in regions called **magnetic domains**. In unmagnetized ferromagnetic material, those domains have random orientations. An external magnetic field causes those domains to grow and/or rotate to give a large magnetization. Figure 28.28 shows a **magnetization curve** for a ferromagnetic material. On the graph, B_0 is the component of the magnetic field that we'd have if no material were present and M is the component of the magnetization \vec{M} in \vec{B}_0 's initial direction. When $M = M_{\text{sat}}$ (where sat is short for *saturation*), the domains are as aligned as possible.

If the domains tend to remain aligned even after the external magnetic field is removed, we have the phenomenon called *hysteresis*. Hysteresis gives us permanent magnets and magnetic memory materials. Figure 28.29 shows a different **hysteresis loop** for each of three applications.