PHYS 242 BLOCK 7 NOTES

Sections 29.1 to 29.7, 30.1 to 30.3

Experimentally, we find $\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt}}$, called **Faraday's law** of induction.

$\mathcal{E}$ is the **induced emf** (in V)—a potential difference that *may* give an **induced current**.

N *is* the number of turns (no unit).

Φ_B is the average magnetic flux (in Wb) through each turn at some time t (in s).

$\frac{d\Phi_B}{dt}$ is the *rate* at which Φ_B is changing with time (in $\frac{\text{Wb}}{\text{s}}$). Thus $1\ \frac{\text{Wb}}{\text{s}} = 1$ V.

Lenz's law: The direction of any magnetic induction effect is such as to oppose the cause of the effect.

The derivation on page 967 gives a relation that we can write more exactly as $\boxed{\mathcal{E} = v_\perp B_\perp l_\perp}$.

$\mathcal{E}$ is the (motional) emf (in V).

$v_\perp$, $B_\perp$, and $l_\perp$ are the *mutually perpendicular components* of the velocity $\vec{v}$ (in $\frac{\text{m}}{\text{s}}$), the uniform magnetic field $\vec{B}$ (in T), and the length vector $\vec{l}$ (in m) of a straight segment.

Cover up the solutions and carefully work Examples 29.1 to 29.5 and 29.7 to 29.10.

In TEST YOUR UNDERSTANDING on page 969, it should say, "The earth's magnetic field points toward the earth's south magnetic pole, which is near the earth's north geographic pole (see Fig. 27.3)."

Consider a single turn ($N = 1$). Since the emf is a potential difference, and a potential difference equals the integral of $\vec{E} \cdot d\vec{l}$, Faraday's law of induction becomes $\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$.

This $\vec{E}$ is the *induced electric field* (in $\frac{\text{V}}{\text{m}}$ or $\frac{\text{N}}{\text{C}}$).

Cover up the solution and carefully work Example 29.11.

Eddy currents are induced currents that circulate within the volume of a *conducting* material.

Trying to apply Ampere's law ($\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$) in Fig. 29.22 gives $I_{\text{encl}} = i_C$ through the plane surface, but $I_{\text{encl}} = 0$ through the bulging surface. Maxwell resolved this contradiction by defining a quantity in the dielectric called the **displacement current** i_D: $\boxed{i_D \equiv \varepsilon\frac{d\Phi_E}{dt}}$. The displacement current is *not* an actual motion of charge, but has the SI unit of ampere = amp = A. Note $i_D = 0$ *only* if the electric flux Φ_E is constant.

ε is the permittivity (in $\frac{\text{C}^2}{\text{N·m}^2}$ or $\frac{\text{F}}{\text{m}}$).

Φ_E is the electric flux (in $\frac{\text{N·m}^2}{\text{C}}$ or V·m).

$\frac{d\Phi_E}{dt}$ is the *rate* at which the electric flux changes with time (in $\frac{\text{N·m}^2}{\text{C·s}}$ or $\frac{\text{V·m}}{\text{s}}$).

Thus, when *no* magnetic materials are present, Ampere's law becomes $\oint \vec{B} \cdot d\vec{l} = \mu_0(i_C + i_D)_{\text{encl}}$.

i_C is the enclosed conduction current (in A) (caused by actual motion of charge)

i_D is the enclosed displacement current (in A) (caused by changing enclosed electric flux).

The four **Maxwell's equations** summarize classical electromagnetism. The first four questions on your *final* exam will involve matching verbal descriptions to these four equations. For no dielectric or magnetic materials, Maxwell's equations reduce to:

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0}$, telling us electric field lines can start on positive charges and end on negative charges.

$\oint \vec{B} \cdot d\vec{A} = 0$, telling us there are evidently no magnetic monopoles on which to start and to end magnetic field lines.

$\oint \vec{B} \cdot d\vec{l} = \mu_0(i_C + \varepsilon_0 \frac{d\Phi_E}{dt})$, telling us closed magnetic field lines are produced by the motion of charge and/or by changing electric flux.

$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$, telling us closed electric field lines are evidently produced only by changing magnetic flux.

Suppose we have two coils, as in Fig. 30.1. A changing current in coil 1 will produce a changing magnetic field, giving changing magnetic flux through coil 2 and, by Faraday's law of induction, an induced emf in coil 2 : $\mathcal{E}_2 = -N_2\frac{d\Phi_{B2}}{dt}$, where $N_2\Phi_{B2} = M_{21}i_1$. Also $\mathcal{E}_1 = -N_1\frac{d\Phi_{B1}}{dt}$, where $N_1\Phi_{B1} = M_{12}i_2$. Advanced electromagnetism courses show $M_{21} = M_{12} = M$. Therefore, $\boxed{\mathcal{E}_2 = -M\frac{di_1}{dt}}$ and $\boxed{\mathcal{E}_1 = -M\frac{di_2}{dt}}$.

$\mathcal{E}_2$ is the emf (in V) induced in coil **2** due *solely* to the time rate of change of current $\frac{di_1}{dt}$ (in $\frac{\text{A}}{\text{s}}$) in coil **1**.

$\mathcal{E}_1$ is the emf (in V) induced in coil **1** due *solely* to the time rate of change of current $\frac{di_2}{dt}$ (in $\frac{\text{A}}{\text{s}}$) in coil **2**.

M is the **mutual inductance** (in henry = H).

Solving both $N_2\Phi_{B2} = Mi_1$ and $N_1\Phi_{B1} = Mi_2$ for M gives us $\boxed{M = \frac{N_2\Phi_{B2}}{i_1} = \frac{N_1\Phi_{B1}}{i_2}}$ .In this equation, all quantities are never negative.

N_2 and N_1 are the numbers of turns in coils 2 and 1 (no units).

Φ_{B2} is the average magnetic flux (in Wb) through each turn of coil **2** due *solely* to i_1, the current (in A) in coil **1**.

Φ_{B1} is the average magnetic flux (in Wb) through each turn of coil **1** due *solely* to i_2, the current (in A) in coil **2**.

The inductance unit is one henry, $1\text{ H} = 1\frac{\text{Wb}}{\text{A}} = 1\frac{\text{T}\cdot\text{m}^2}{\text{A}} = 1\frac{(\text{N/A}\cdot\text{m})\cdot\text{m}^2}{\text{A}} = 1\frac{\text{J}}{\text{A}^2} = 1\frac{\text{J}}{(\text{C/s})\cdot\text{A}} = 1\frac{\text{V}\cdot\text{s}}{\text{A}} = 1\ \Omega\cdot\text{s}$.

Cover up the solutions and carefully work Examples 30.1 and 30.2.

Now suppose we consider a single coil, as in Fig. 30.4. A changing current in that coil will produce a changing magnetic field, giving changing magnetic flux through that coil and, by Faraday's law of induction, an self-induced emf $\mathcal{E}$ (in V): $\mathcal{E} = -N\frac{d\Phi_B}{dt}$, where $N\Phi_B = Li$, which gives $\boxed{\mathcal{E} = -L\frac{di}{dt}}$. Thus $\boxed{L = \frac{N\Phi_B}{i}}$ is the **self-inductance** L (in H) of that coil (often just called the *inductance* of that coil). In $L = \frac{N\Phi_B}{i}$, all four quantities are never negative.

Cover up the solutions and carefully work Examples 30.3 and 30.4.

Suppose we have two ordinary coils. Each coil will have a self-inductance (L_1 and L_2) and the two coils may also have a mutual inductance (M). Thus, for coil **1**, we can write $\mathcal{E}_{1,\text{self}} = -L_1\frac{di_1}{dt}$, with $L_1 = \frac{N_1(\Phi_B)_1}{i_1}$, where $(\Phi_B)_1$ is the average magnetic flux through each of the N_1 turns of coil **1** due *solely* to coil **1**'s current i_1. Thus $(\Phi_B)_1$ (from coil **1**'s current) is *not* the same average magnetic flux as Φ_{B1} (from coil **2**'s current). For coil **2**, we similarly write $\mathcal{E}_{2,\text{self}} = -L_2\frac{di_2}{dt}$, with $L_2 = \frac{N_2(\Phi_B)_2}{i_2}$ [where $(\Phi_B)_2$ (self) is *not* the same as Φ_{B2} (mutual)]. That is, we have three possible inductances (L_1,L_2, and M) arising from four possible average magnetic fluxes [$(\Phi_B)_1$, $(\Phi_B)_2$, Φ_{B1}, and Φ_{B2}]. Then, changes in those average magnetic fluxes with time give four emfs ($\mathcal{E}_{1,\text{self}}$, $\mathcal{E}_{1,\text{mutual}}$, $\mathcal{E}_{2,\text{self}}$, and $\mathcal{E}_{2,\text{mutual}}$).

An **inductor** (sometimes called a *choke*) is a circuit element used mainly for its inductance.

On page 998, the text derives an expression for the magnetic potential energy U (in J) stored in the magnetic field of an inductor of (self-)inductance L (in H) when carrying a current I (in A): $\boxed{U = \frac{1}{2}LI^2}$. The unit equivalence $1\text{ H} = 1\ \frac{\text{J}}{\text{A}^2}$ is helpful in this equation.

Cover up the solution and carefully work Example 30.5.

We used $U = \frac{1}{2}\ CV^2$ and a parallel-plate capacitor to find the energy density u of an electric field. Similarly, we now use $U = \frac{1}{2}\ LI^2$ and a toroid to find the energy density u of a magnetic field. Consider a toroid of small cross section and two layers of wire wound so that its magnetic field is zero outside the "dough" of its doughnut-shaped core.

First we find its self-inductance $L = \frac{N\Phi_B}{i} = \frac{NBA}{i}$. We substitute $B = \frac{\mu NI}{2\pi r}$ and $i = I$ to find $L = \frac{\mu N^2 A}{2\pi r}$. By definition, $u = \frac{U}{\text{volume}} = \frac{\frac{1}{2}LI^2}{\text{volume}} = \frac{\frac{1}{2}\frac{\mu N^2 A}{2\pi r}I^2}{(2\pi r)A} = \frac{1}{2\mu}\left(\frac{\mu NI}{2\pi r}\right)^2 = \frac{1}{2\mu}\ B^2$. This final expression contains none of the dimensions of the toroid and is true for all linear materials (materials with a constant μ): $\boxed{u = \frac{B^2}{2\mu}}$.

u is the energy density (in $\frac{\text{J}}{\text{m}^3}$) of the magnetic field $\vec{B}$ (in T).

μ (mu) is the **permeability** of the material (in $\frac{\text{T·m}}{\text{A}}$).

μ_0, the magnetic constant, is also the permeability of vacuum ($\mu_0 \equiv 4\pi \times 10^{-7}\ \frac{\text{T·m}}{\text{A}}$).

Thus $\mu \equiv \mu_0$ by definition for vacuum and also for nonmagnetic materials. Because of their ordinarily weak magnetizations, μ is slightly greater than μ_0 for paramagnetic materials (if not at very low temperatures) and μ is slightly less than μ_0 for ordinary (not superconducting) diamagnetic materials.

We *must* distinguish between U (magnetic potential energy), u (energy density), and μ (permeability). To minimize confusion, in this block we will *not* also use μ to stand for the magnitude of the magnetic dipole moment (as in $\mu = NIA$). However, we may use the metric prefix $\mu \equiv 10^{-6}$, as in μH.