PHYS 242 BLOCK 9 NOTES

Sections 15.8, 16.1 to 16.5, 16.7

Let's again consider a standing wave on a string (or wire or cable or rope or spring or …) that is *fixed at both ends* (of its vibrating portion). A **normal mode** is a motion in which all the moving particles of the medium oscillate sinusoidally at the *same* frequency f. For a normal mode: 1. the nodes and antinodes alternate, 2. the distance from a node to its nearest antinode is one-quarter wavelength ($\frac{\lambda}{4}$) and, 3. the distance from a node to its nearest node (or from an antinode to its nearest antinode) (if they exist) is one-half wavelength ($\frac{\lambda}{2}$).

The fixed ends are nodes, so the vibrating length L must contain a whole number of half-wavelengths. That is, $L = n\frac{\lambda_n}{2}$ ($n = 1, 2, 3, 4, \ldots$). Thus the *allowed* wavelengths are $\lambda_n = \frac{2L}{n}$. Using $\boxed{v = \lambda_n f_n}$, we find the *allowed* frequencies: $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ in particular and $f_n = \frac{v}{\lambda_n} = \frac{v}{2L/n} = n\frac{v}{2L} = nf_1$ ($n = 1, 2, 3, 4, \ldots$) in general.

From Block 8, recall $v = \sqrt{\frac{F}{\mu}}$, so that $\boxed{f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}}$.

F is the magnitude of the tension (in N) in the string ….

v is the wave speed (in $\frac{\text{m}}{\text{s}}$) of the two traveling waves on the string … that make up the standing wave.

L is the length (in m) between the fixed ends of the vibrating portion of the string ….

μ is the mass per length, $\frac{m}{L}$ (in $\frac{\text{kg}}{\text{m}}$, NOT $\frac{\text{g}}{\text{m}}$) of the string ….

f_1 (in Hz) is the first **harmonic**, or the **fundamental frequency** (the lowest allowed frequency).

f_2 is the 2nd harmonic and the 1st **overtone** (i. e., the first *tone*, or frequency, *over* the fundamental).

f_3 is the 3rd harmonic and the 2nd overtone, and so on.

Thus, n is the harmonic number. Also, n gives the number of half-wavelengths as well as the number of "loops" in the standing wave (see Figs. 15.23 and 15.26).

Cover up the solutions and carefully work Examples 15.7 and 15.8.

A **sound** wave in air is a longitudinal pressure wave. The range of frequencies humans can hear varies from person to person and is somewhat arbitrarily set to be from 20 Hz to 20,000 Hz. Then **ultrasonic** sound waves have frequencies *above 20,000 Hz* and **infrasonic** sound waves have frequencies *below 20 Hz*.

In an ideal gas, $\boxed{v = \sqrt{\frac{\gamma RT}{M}}}$. The quantities in this equation are never negative.

v is the speed (in $\frac{\text{m}}{\text{s}}$) of sound in an ideal gas (a gas with $pV = nRT$ as its equation of state).

γ (gamma) (no unit) is the molar specific heat capacity ratio C_p/C_V defined on page 633.

R is the gas constant ($R = 8.3145\ \frac{\text{J}}{\text{mol·K}}$) and T is the Kelvin temperature (in K).

M is the molar mass (in $\frac{\text{kg}}{\text{mol}}$, NOT $\frac{\text{g}}{\text{mol}}$) (recall that 1 g = 10^{-3} kg).

Carefully work Example 16.4 to show the speed of sound in air is 344 m/s at 20˚C (= 68˚F = 293 K).

The **intensity** I (in $\frac{\text{W}}{\text{m}^2}$) of a wave is the average power P_{av} (in W) it transmits per perpendicular area (in m^2). Example 16.7 makes some not-too-realistic assumptions in applying this idea to a damaging intensity.

We do not hear sounds of intensity $10I_1$ and $100I_1$ as 10 and 100 times as loud as a sound of intensity I_1, but roughly two and three times as loud. Therefore, we define the logarithmic **sound intensity level** β (beta) (in dB = decibels) by $\boxed{\beta \equiv (10\text{ dB}) \log \frac{I}{I_0}, \text{ where } I_0 \equiv 10^{-12}\,\frac{\text{W}}{\text{m}^2}}$. (Often, β is called just the *sound level*.)

I is the sound intensity (in $\frac{\text{W}}{\text{m}^2}$): few people can hear a sound of intensity less than I_0 (*defined* to equal $10^{-12}\,\frac{\text{W}}{\text{m}^2}$).

To find I when given β, we first divide both sides of the above equation by (10 dB), then take the antilog of both sides, then multiply through by I_0, giving $\boxed{I = I_0\, 10^{(\beta/10\text{ dB})}}$.

Cover up the solution and carefully work Example 16.8.

In a pipe, two sound waves that are identical except for opposite velocities set up a standing wave in that pipe. Such a standing wave will have a *displacement node* (N) at a pipe's stopped (closed) end and a *displacement antinode* (A) slightly outside any open end. In this introductory course, we will ignore actual *end correction(s)*, and will always place the displacement antinode(s) *at* the pipe's open end(s).

For standing sound waves: a displacement node is always a pressure antinode and a displacement antinode is always a pressure node .

An *open pipe* is one that is open at *both* ends. For normal modes, an open pipe must have a displacement antinode (A) at each end. Thus its length L must contain a whole number of half-wavelengths—the same condition that held for the string … fixed at both ends—so we see $\boxed{L = n\frac{\lambda_n}{2} \text{ or } \lambda_n = \frac{2L}{n}\ (n = 1, 2, 3, 4, \ldots)}$ and $\boxed{f_1 = \frac{v}{2L} \text{ and } f_n = n\frac{v}{2L} \text{ or } f_n = nf_1\ (n = 1, 2, 3, 4, \ldots)}$ hold for standing waves *both* on a string … fixed at both ends *and* in an open pipe.

A *stopped* pipe is one that is stopped (that is, closed) at *one* end but open at the other end. For normal modes, a stopped pipe must have a displacement node (N) at the stopped end and a displacement antinode (A) at the open end. The distance from a node to an adjacent antinode is a quarter-wavelength, so the length L must contain an odd number of quarter-wavelengths: $\boxed{L = n\frac{\lambda_n}{4} \text{ or } \lambda_n = \frac{4L}{n}\ (n = 1, 3, 5, 7, \ldots)}$. From $f_n = \frac{v}{\lambda_n}$, we have $\boxed{f_1 = \frac{v}{4L} \text{ and } f_n = n\frac{v}{4L} \text{ or } f_n = nf_1\ (n = 1, 3, 5, 7, \ldots)}$. The condition ($n = 1, 3, 5, 7, \ldots$) tells us **a stopped pipe has no even harmonics**. For a stopped pipe, the fundamental frequency still has $n = 1$, but the first overtone has $n = 3$, the second overtone has $n = 5$, the third overtone has $n = 7$, and so on. (Thus, the fourth overtone is the ninth harmonic.) Cover up the solution and carefully work Example 16.11.

A **forced oscillation** is a vibration caused by an externally applied force (or torque, …). In the important phenomenon called **resonance**, a large response occurs when the frequency of the forced oscillation matches the frequency of a normal mode of a system. Carefully work through Example 16.12.

The interference of two *equal*-frequency waves *in space* gave us a standing wave. The interference of two *different*-frequency sound waves *in time* causes **beats**, which are regular variations in the loudness of the sound. Section 16.7 in our text shows $\boxed{f_{\text{beat}} = f_a - f_b}$, where $f_a > f_b$ and none of the frequencies are negative.

f_{beat} is the beat frequency (in beats/s or Hz), while f_a and f_b are the two different frequencies (both in Hz). Try the Test Your Understanding of Section 16.7 question (page 528). Its answer is explained on page 544.