

PHYS 242 BLOCK 11 NOTES

Sections 33.1 to 33.7

Geometrical Optics

At all points on a **wave front**, the wave has the same phase of oscillation, e. g., a crest. (illustrated in Figs. 33.3 and 33.4).

Strictly speaking, a **ray** is an imaginary line drawn along the direction of travel of the waves. If the material has the same optical properties in all regions (is homogenous) and in all directions (is isotropic), the rays in the material are *straight lines* perpendicular to the wave fronts as illustrated in Fig. 33.4.

**Reflection** is the “bouncing off” of a ray from a surface. For reflection from a smooth surface (specular reflection, not diffuse reflection),  $\theta_r = \theta_a$ . See Figs. 33.5c, 33.6, 33.7, and several others.

$\theta_a$  is the **angle of incidence**—the angle the incident ray makes *with the normal to the surface*.

$\theta_r$  is the **angle of reflection**—the angle the reflected ray makes *with the normal to the surface*.

These two rays are on opposite sides of the normal. These two rays and their normal are in the same plane.

**Refraction** is the transmission of a wave across the boundary *from* optical material *a* to optical material *b*. (Since *a* is the *first* letter of our alphabet, material *a* is where the light is *first* found.)

$\theta_b$  is the **angle of refraction**—the angle the refracted ray makes *with the normal to the surface*. The refracted ray does NOT bounce off the normal (see Figs. 33.5c, 33.7, 33.8, and several others). The refracted ray is in the same plane as the incident ray, the reflected ray, and their normal.

An optical material’s **index of refraction**  $n$  is defined by the equation  $n \equiv \frac{c}{v}$ .

$c$  is the speed of light in vacuum, defined to equal exactly 299,792,458 m/s  $\approx 2.998 \times 10^8$  m/s.

$v$  is the speed of light in the material (in m/s) and  $v \leq c$ .

Thus  $n$  has no unit,  $n \geq 1$ ,  $n = \sqrt{K K_m}$  (from Block 10),  $n = 1$  in vacuum, and  $n \approx 1$  in air.

Traditionally called **Snell’s law**, the law of refraction is  $n_a \sin \theta_a = n_b \sin \theta_b$ . Unless  $\theta_a = 0 = \theta_b$ , light bends toward the normal when it slows down (when  $v$  decreases,  $n$  increases so  $\theta$  decreases, Fig. 33.8a) and light bends away from the normal when it speeds up (when  $v$  increases,  $n$  decreases so  $\theta$  increases, Fig. 33.8b). (The right-angle symbols in Fig. 33.8c do not mean  $\theta_a$  and  $\theta_b$  are  $90^\circ$ —rather, the incident and refracted rays are both perpendicular to the boundary, which makes the rays parallel to the normal. Thus, in that figure,  $\theta_a$  and  $\theta_b$  are zero for any values of  $n_a$  and  $n_b$ .)

Refraction across a boundary does not create or absorb waves each second, so refraction does not change the frequency  $f$ . In vacuum,  $c = \lambda_0 f$ , and in the material,  $v = \frac{c}{n} = \lambda f$ . Solving to eliminate  $f$ , we obtain  $\lambda = \frac{\lambda_0}{n}$ ,

where  $\lambda_0$  is the wavelength in vacuum and  $\lambda$  is the wavelength in a material that has an index of refraction  $n$ . Since  $n \geq 1$ ,  $\lambda \leq \lambda_0$ .

For transparent media, *some* of the incident light reflects whenever the index of refraction changes. However, for any angle of incidence greater than or equal to the **critical angle**  $\theta_{\text{crit}}$ , *all* of the incident light reflects (therefore, none refracts), a phenomenon called **total internal reflection**. See Figure 33.13a. When  $\theta_a = \theta_{\text{crit}}$ ,  $\theta_b = 90^\circ$ . Therefore, Snell's law gives  $n_a \sin \theta_{\text{crit}} = n_b \sin 90^\circ = n_b (1)$ . Solving,  $\sin \theta_{\text{crit}} = \frac{n_b}{n_a}$ . Because the sine of an angle cannot be greater than one,  $n_b$  must be less than  $n_a$ . That is, **total internal reflection occurs only for reflection off a material of lower index of refraction**.

As illustrated in Fig. 33.17, we call the dependence of  $n$  on  $\lambda_0$  **dispersion** (because, as illustrated in Figs. 33.18 and 33.19, this dependence can cause the dispersion of white light into a spectrum).

So-called “unpolarized light” or “natural light” has a random mixture of linear polarizations. An ideal **polarizer** transmits only those  $\vec{E}$  components of an em wave that oscillate parallel to its **polarizing axis**. An **analyzer** is simply a second polarizer. Recalling from Block 10 that the intensity of an em wave is proportional to  $E_{\text{max}}^2$ , Fig. 33.24 shows  $I = I_{\text{max}} \cos^2 \phi$ .

$I_{\text{max}}$  is the intensity (in  $\frac{\text{W}}{\text{m}^2}$ ) of the light *after* passing through the ideal polarizer:  $I_{\text{max}} = \frac{1}{2} I_0$  if the original incident light (of intensity  $I_0 = I_{\text{original}}$ ) is “unpolarized”.

$I$  is the intensity (in  $\frac{\text{W}}{\text{m}^2}$ ) of the light *after* passing through the ideal analyzer.

$\phi$  is the angle between the polarizing axes of the polarizer and the analyzer.

Experimentally (or from Maxwell's equations): When  $\theta_a + \theta_b = 90^\circ$ , the *reflected* light is 100% linearly polarized (with its electric field vector parallel to the surface). Then we call  $\theta_a$  the **polarizing angle**  $\theta_p$ . Applying Snell's law,  $n_a \sin \theta_p = n_b \sin (90^\circ - \theta_p) = n_b \cos \theta_p$ . Since  $\frac{\sin \theta_p}{\cos \theta_p} = \tan \theta_p$ , we find  $\tan \theta_p = \frac{n_b}{n_a}$ .

Thus the angle of incidence  $\theta_a$  may have two special values: the critical angle  $\theta_{\text{crit}}$  and the polarizing angle  $\theta_p$ .

We can construct new wave fronts from old using **Huygens' principle**. As illustrated in Fig. 33.33:

- 1) All points of a wave front are considered as sources of secondary wavelets that spread out at wave speed  $v$ .
- 2) A new wave front is then tangent to those wavelets at time  $t$ .

Figures 33.34 and 33.35 use Huygens' principle to derive the law of reflection and Snell's law (the law of refraction). Those two laws can also be derived using Maxwell's equations.