

PHYS 242 BLOCK 13 NOTES

Sections 34.7, 34.8, 35.1 to 35.5

A **magnifier** is a converging lens used with a real object placed at or inside its first focal point. Then the magnifier gives a virtual, erect, enlarged image that has an **angular magnification**  $M$ . Note that the *angular* magnification  $M$  is *not* the same thing as the *lateral* magnification  $m$  discussed in Block 12.

Let 25 cm be the nearest distance at which an ordinary human eye can focus. As in Fig. 34.51a, first place the real object 25 cm from the eye to find  $\theta$ . Then place the object in the focal plane of the converging lens ( $s = f$ ).

Finally, look through the lens to find  $\theta'$  (Fig. 34.51b). For small angles,  $M = \frac{\theta'}{\theta} = \frac{25 \text{ cm}}{f}$ .  $M$  has no unit.

$\theta$  is the angle subtended at the eye by the object (25 cm from the eye) when the magnifier is *not* present.

$\theta'$  is the angle subtended at the eye by the image seen through the magnifier (when the object has  $s = f$ ).

$f$  is the positive focal length (in cm) of the converging lens.

In a **microscope**, the real image from the first lens acts as a real object for the second lens, giving a large net angular magnification of *nearby* objects. (See Fig. 34.52b.)

In a refracting **telescope**, the same process occurs, except for *distant* objects. (see Fig. 34.53.)

Physical Optics

**Monochromatic light** is light of a single frequency  $f$  (and a single vacuum wavelength  $\lambda_0$ ). Monochromatic light is an ideal approached by laser light.

**Coherence** means having a definite constant phase relation.

First consider two coherent sources emitting, in phase, monochromatic waves that have the same  $f$  (and thus the same  $\lambda$ ). Maximum **constructive interference** occurs when the waves arrive at a point in phase. Since each wave is like the others preceding and following it, we can say the waves arrive “in phase” when their **phase difference**  $\phi$  equals  $\pm 2\pi, \pm 4\pi, \dots$  radians as well as zero, that is, when  $\phi = m(2\pi)$  with  $m = 0, \pm 1, \pm 2, \pm 3, \dots$ . Also maximum **destructive interference** occurs when the waves arrive  $\pm\pi, \pm 3\pi, \pm 5\pi, \dots$  rad out of phase.

If the waves from such coherent sources 1 and 2 travel path lengths of  $r_1$  and  $r_2$  from those sources to the point where they interfere,  $r_2 - r_1$  is their **path difference**. Suppose the phase difference  $\phi$  is due solely to the path difference  $r_2 - r_1$ . Then a little thought should tell you that the ratio of the path difference to the wavelength  $\lambda$  (both in the same unit) equals the ratio of the phase difference  $\phi$  to  $2\pi$  rad. For example, a path difference of 2.5 wavelengths ( $2.5\lambda$ ) gives a phase difference of 2.5 times  $2\pi$  rad.

Mathematically equating the ratios:  $\frac{r_2 - r_1}{\lambda} = \frac{\phi}{2\pi}$ , which solves to  $\phi = \frac{2\pi}{\lambda} (r_2 - r_1)$ .

Since the “ $2\pi$ ” in the boxed equation above is  $2\pi$  rad,  $\phi$  is in rad.

You can replace the  $2\pi$  rad with  $360^\circ$  if you want—or are given—the phase difference  $\phi$  in degrees.

Suppose our coherent sources are two (or more) narrow rectangular slits (see Fig. 35.5), that the center-to-center distance between adjacent slits is  $d$ , and that the distance from the slits to a screen is much greater than either  $\lambda$  or  $d$ . Corrected Fig. 35.5c shows the path difference  $r_2 - r_1$  equals  $d \sin \theta$  for adjacent slits.

Substituting  $r_2 - r_1 = d \sin \theta$  and  $\phi = m(2\pi)$  in the preceding boxed equation for the phase difference  $\phi$  gives  $d \sin \theta = m\lambda$  ( $m = 0, \pm 1, \pm 2, \pm 3, \dots$ ) as the relation for *constructive interference* for two (or more) rectangular slits. This constructive interference will give the bright fringes of light illustrated in Fig. 35.5a and shown in Fig. 35.6.

The “ $m$ ” in this equation is *not* the lateral magnification and *not* the mass, but is an integer.

As usual,  $\theta$  is the angle the rays make with the normal.

Since  $d$  and  $\lambda$  are intrinsically positive, we see that  $m$  and  $\theta$  *must* have the same sign.

Example 35.2 applies this equation to radio waves.

We now consider interference effects caused by light reflected from the two sides of a thin insulating transparent film of thickness  $t$  surrounded by dielectrics. For simplicity, we consider only normal (perpendicular) incidence. (However, we’ll draw the rays at an angle with the normal for clarity.) Maxwell’s equations give Eq. (35.16), where the  $E$ ’s are *components*, not amplitudes. This equation tells us the light undergoes:

**a zero phase shift upon reflection off a lower index of refraction medium ( $n_b < n_a$ ), but**

**a  $\pi$  rad phase shift upon reflection off a higher index of refraction medium ( $n_b > n_a$ ).**

This equation also tells us that *no* light is reflected if there is *no* change in index of refraction ( $n_b = n_a$ ).

There is also a phase difference between the two reflected rays caused by their path difference  $2t$ . Thus we have  $2t = m\lambda$  with  $m = 1, 2, 3, \dots$  or  $2t = (m + \frac{1}{2})\lambda$  with  $m = 0, 1, 2, 3, \dots$ . In either case,  $\lambda = \frac{\lambda_0}{n}$ .

$\lambda$  is the wavelength *in the film*,  $n$  is the index of refraction *of the film*, and  $t$  is the thickness *of the film*.

$\lambda_0$  is the wavelength *in vacuum*.

For a given value of  $\lambda$  or  $\lambda_0$ , the smallest allowed value of  $m$  gives the smallest thickness  $t$ .

**By conservation of energy, a reflection *maximum* gives a transmission *minimum* and a reflection *minimum* gives a transmission *maximum*.**

Which boxed “ $2t = \dots$ ” equation above should we use? First we must determine the phase shifts upon reflection off one side of the film and also off the other side of the film. (Those phase shifts can be both zero, both  $\pi$  rad, or one zero and the other  $\pi$  rad.) Then use logic or see the SUMMARY below (also found on your Objectives sheet).

#### SUMMARY—THIN INSULATING FILMS, NORMAL INCIDENCE

Number of phase shifts of $\pi$ rad upon reflection	zero or two	one
Reflection maximum and transmission minimum	$2t = m\lambda$	$2t = (m + \frac{1}{2})\lambda$
Reflection minimum and transmission maximum	$2t = (m + \frac{1}{2})\lambda$	$2t = m\lambda$

Figure 35.19 represents a **Michelson interferometer**. If you move the movable mirror a distance  $y$ , the path difference changes by  $2y$  [ $y_{\text{out}}$  plus  $y_{\text{back}}$ ]. Changing the path difference changes the interference pattern—with either  $m$  bright fringes or  $m$  dark fringes moving past a point in that pattern when  $2y = m\lambda$ .