PHYS 242 BLOCK 14 NOTES

Sections 36.1 to 36.8

Diffraction is the interference phenomenon that occurs when many waves are combined. For example, diffraction occurs when waves strike a barrier that has an edge or when waves pass through an opening (aperture). The constructive and destructive interference of these many waves gives **diffraction patterns**.

Fresnel diffraction, also known as **near-field diffraction**, occurs when the distances from the source and screen to the barrier are *not* much greater than the wavelength λ.

We will consider **Fraunhofer diffraction**, also known as **far-field diffraction**, which occurs when the distances from the source and screen to the barrier *are* much greater than the wavelength λ.

For a single rectangular slit of width a, $\boxed{\sin\,\theta = m\frac{\lambda}{a}\ (m = \pm 1, \pm 2, \pm 3, \ldots)}$ is the condition for *destructive* interference in its diffraction pattern. As usual, θ is the angle with the normal to the slit. This destructive interference will give the dark fringes—the diffraction zeroes—shown photographically in Fig. 36.6. Note that m and θ have the same sign. The allowed values of m do *not* include $m = 0$ because $m = 0$ gives maximum *constructive* interference at $\theta = 0$ (the center of the *brightest fringe* in the slit's diffraction pattern).

Figure 36.7 is an artist's version of Fig. 36.6. Can you improve of the artist's version? One change would to make it red light (633 nm) instead of blue light.

Now we consider the intensity of the light from two identical rectangular slits. Ignoring the diffraction pattern, the text derives Eq. (35.10) for the interference pattern (graphed in Figs. 35.10 and 36.12b). Later, the text derives Eq. (36.5) for the diffraction pattern of each slit (graphed in Fig. 36.12a). Combining the two equations gives us $\boxed{I = I_0 \cos^2\frac{\phi}{2}\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2}$, which is graphed in Fig. 36.12c and shown photographically in Fig. 36.12d for $d = 4a$.

In the boxed equation above, **the β in the denominator *must* be in radians** (because the derivation uses $\beta \equiv \frac{\text{arc length}}{\text{radius}}$, which *defines* an angle in radians).

I is the intensity (in $\frac{\text{W}}{\text{m}^2}$) at an angle θ with the normal.

I_0 is the maximum intensity (in $\frac{\text{W}}{\text{m}^2}$), which occurs directly ahead where seven quantities equal zero. The seven quantities are θ, ϕ, β, both $(r_2 - r_1)$'s, and both m's.

ϕ is the phase difference for light from the *same* part of the two *separate* slits (for example, for rays from the center of one slit and from the center of the other slit). Thus $\phi = \frac{2\pi}{\lambda}\,d\,\sin\,\theta$.

β is the phase difference for light from the top and from the bottom of the *same* slit. Thus $\beta = \frac{2\pi}{\lambda}\,a\,\sin\,\theta$.

Since d must be larger than a, ϕ must be larger than β by the same ratio. For example, if $d = 4a$, $\phi = 4\beta$.

A **diffraction grating** is an array of a large number (N) of rectangular slits. Each slit has a width a. The center-to-center distance between adjacent slits is d. If $\boxed{d \sin \theta = m\lambda \ (m = 0, \pm 1, \pm 2, \pm 3, \ldots)}$, we get very sharp constructive interference maximums. We call $|m|$ the **order number**. For instance, in the fourth **order**, $m = +4$ gives a positive angle θ with the normal and $m = -4$ gives a negative angle θ (if they exist).

Understand that 1/d = # slits/distance and so $d = \dfrac{1}{\text{\# slits/distance}}$. For example, for a diffraction grating that has 500 slits/mm, $d = \dfrac{1}{500/(10^{-3}\text{ m})} = \dfrac{10^{-3}\text{ m}}{500} = 2.00 \times 10^{-6}$ m = 2.00 μm = 2000 nm.

Missing orders occur when the angle expected for an interference maximum ($\theta_{\text{int max}} = \arcsin \dfrac{m_{\text{int}}\lambda}{d}$) is the same as the angle for a diffraction zero ($\theta_{\text{diff zero}} = \arcsin \dfrac{m_{\text{diff}}\lambda}{a}$). Equating the angles gives $\dfrac{m_{\text{int}}}{d} = \dfrac{m_{\text{diff}}}{a}$. For example, if $d = 4a$, every fourth non-zero order would be missing (if they exist), as shown in Fig. 36.12.

Suppose we have two wavelengths of average wavelength λ, separated by $\Delta\lambda$ ($\Delta\lambda = \lambda_{\text{high}} - \lambda_{\text{low}}$). The **chromatic resolving power** R (no unit—*not* an actual power in watts) is defined to be the ratio of λ to $\Delta\lambda$. Pages 1200 and 1201 show that, for a diffraction grating of N slits in the $|m|^{\text{th}}$ order, $\boxed{R \equiv \dfrac{\lambda}{\Delta\lambda} = N|m|}$.

In **x-ray diffraction**, we see diffraction patterns from the interference of reflections from many atoms in a crystal (see Figs. 36.20b and 36.24).

In cameras, telescopes, microscopes, our eyes, etc., we pass waves of wavelength λ through a circular aperture (round opening) of diameter D (rather than through a rectangular slit). If so, advanced analysis shows the first diffraction zero occurs where $\boxed{\sin \theta_1 = 1.22 \dfrac{\lambda}{D}}$.

Suppose we have two point objects separated by a transverse distance y_1 that are a distance s from a circular aperture. Then $\boxed{\tan \theta_1 = \dfrac{y_1}{s}}$. According to **Rayleigh's criterion**, the two objects will be barely resolved in angular position if $\theta_1 = \arcsin 1.22 \dfrac{\lambda}{D}$. (See the images of sources 3 and 4 in Fig. 36.27a.) Thus, θ_1, the angle with the normal for the first diffraction zero, is also the angular separation of the two barely resolved point objects.

Why use large-aperture telescopes? They have a large diameter D. Therefore,

1) their larger area ($= \dfrac{\pi D^2}{4}$) collects more light and thus gives brighter images and

2) their larger D gives better spatial resolution (that is, gives smaller θ_1 and thus smaller y_1).

Figure 36.28a represents the production of a *hologram*. A **hologram** is an interference pattern on photographic film that, when properly illuminated, will produce a three-dimensional image of the original object.